Behavior of Visco-elastic fluid in presence of diffusion of chemically reactive species

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Abstract:

we studied the influence of reaction rate parameter on the transfer of chemically reactive species in the laminar visco-elastic fluid flow for non-linearly stretching surface and the reactive species is emitted from this sheet and under goes an isothermal and homogeneous one-stage reaction as it diffuses in to the surrounding fluid. The important findings of our study are the thickness of the concentration boundary layer decreases with the increasing values of Schmidt number and if the values of Schmidt number increase, reaction rate parameter, wall concentration gradient $\phi_{\eta}(0)$ decreases.

Keywords: Schmidt number Sc, concentration profile $\phi(\eta)$, diffusivity D.

1. Introduction:

Study of heat transfer, mass transfer and momentum transfer in a laminar boundary layer over a moving stretching surface has gained considerably practical relevance in the field of electrochemistry (Chin [1975], Gorla [1978]) and polymer processing (Griffith [1964], Erickson et al. [1966]). The important studies of these transport processes have so far been devoted to flows induced by surfaces moving with constant velocity. Pioneering work was carried out by Sakiadis [1966] and that was extended by Crane [1970]. Due to the increasing applications of non-Newtonian fluids in industries, several researchers (Carragher and Crane [1982], Vleggaar [1977] and Soundalgekar and Murty [1980]) studied the heat transfer problem associated with the Newtonian/non-Newtonian boundary layer flow past a stretching sheet. In these studies there exists a mathematical equivalence of the heat transfer problem with the mass transfer in the boundary layer. Hence, the results obtained for heat transfer characteristics can be carried directly to the mass transfer by replacing Prandl number by Schmidt number. However, the presence of chemical reaction term in the mass diffusion equation generally destroys the formal equivalence with the thermal energy problem. It generally destroys the construction of the otherwise attractive similarity transformations. Chamber and Young [1958] considers diffusion of a chemically reactive species in a fluid past a wedge shaped body and found the existence of a similarity solution only in the case of stagnation point flow. In this regard some studies were carried out for mass transfer in a flow over inclined plates, in the absence of the mass diffusion, (Dural and Hines [1990], Anderson et al.[1994]). But existing literature fails to report any analytical or experimental study on combined diffusion of heat and chemical species over an inclined plate with variable surface temperature. There are flows in which the driving force in the flow is provided by circumstances that arise in many practical situations like, driving cleaning operations where residual fluid diffuses in to the surrounding fluid at different temperature and in the curing of plastics and also in the manufacture of pulp insulated cables.

The physical situation discussed in all the above studies is related to the process of linearly stretching sheet case. Another physical phenomenon is the case in which the sheet is stretched in a non-linear fashion. Gupta and Gupta [1977] have underlined that the stretching of the sheet may not necessarily be linear. In view of this, the nonlinearly stretching sheet was investigated by Vajravelu [1991]. Hence, it is interesting to study the flow and mass transfer phenomenon over a non- linearly stretching sheet. In the present chapter, we study flow and mass transfer on a non-linear stretching sheet with velocity $u_w(x)$ for two different types of thermal boundary conditions on the sheets. Another effect which bear great importance on the heat transfer is the viscous dissipation are also included in the energy equation.

2. Flow Analysis:

We consider the flow of an incompressible visco-elastic fluid past a flat sheet coinciding with the plan y=0, the flow being confined to y>0.Two equal and opposite forces are applied along the x-axis so that the wall is stretched keeping the origin fixed. The steady two dimensional boundary layer equations for this fluid, in the usual notation, are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\gamma\frac{\partial^2 u}{\partial y^2}-k_0\left\{u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right\} \qquad (2)$$

Where (x, y) denotes the Cartesian coordinates along the sheets and normal to it, u and v are the velocity components of the fluid in the x and y directions respectively, and γ is the kinematic viscosity. The boundary conditions for the present problem are

$$u_{w(x)}=\frac{\upsilon}{L^{\frac{4}{3}}}x^{\frac{1}{3}},\quad v=0 \quad y=0$$

$$u\text{-->}0 \quad \text{as} \quad y\text{-->}\infty \qquad (3)$$

Where L is the reference length.

Defining new similarity variables

$$\eta=y\frac{x^{\frac{-1}{3}}}{L^{\frac{2}{3}}},u=\frac{\upsilon}{L^{\frac{4}{3}}}x^{\frac{1}{3}}f'(\eta),v=\frac{\upsilon}{L^{\frac{2}{3}}}x^{\frac{-1}{3}}\frac{(2f-\eta f')}{3} \qquad (4)$$

With these changes of variables, equation (1) is identically satisfied and equation (2) is transformed into the following non-linear ordinary differential equation

$$f'^2-ff''=f'''-k_1\{2f'f'''-ff''''-f''^2\} \qquad (5)$$

and the boundary conditions (3) becomes

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$$f = 0 \quad f' = 1 \qquad at \quad \eta = 0$$

$$f' \to 0 \quad f'' \to 0 \quad \text{as} \quad \eta \to \infty \qquad (6)$$

The shear stress at the stretched surface is defined as

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_w \qquad (7)$$

and we obtain from (4) and (7)

$$\tau_w = \mu \frac{\upsilon}{L^2} f''(0) \qquad (8)$$

Where μ is the viscosity of the fluid.

Problem {(5)-(6)} is solved numerically by employing a Runge-kutta algorithm for higher order initial value problems. Based on the numerical solution, we obtained, $f''(0)$ = -1.289747

3. **Solution of the Concentration Boundary Layer Equation:**

The concentration field C (x,y) is governed by the boundary layer mass diffusion equation

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \qquad (9)$$

Where, C being the concentration of the species of the fluid, D is the diffusion coefficient of diffusion of species in the flow. Since the concentration of the reactant is maintained at a prescribed value C_w at the sheet and it is assumed to vanish far away from the sheet, the relevant boundary conditions for the concentration equation (9) become

$$C = C_w \qquad at \qquad y=0$$

$$C \to C_\infty \qquad \text{as} \qquad y \to \infty \qquad (10)$$

Defining non-dimensional concentration variable as

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

Where $C_w = C_\infty + A \left(\frac{x}{l} \right)^2$ (11)

Using the transformations given by the equations (4) and (11) in equation (9) we get

$$\phi'' + \left(\frac{2}{3} \right) Scf\phi' - 2Scf'\phi = 0 \qquad (12)$$

Where $Sc = \frac{\upsilon}{D}$ is the Schmidt number.

The boundary conditions become

$$\phi = 1 \qquad at \quad \eta = 0$$

$$\phi = 0 \qquad as \quad \eta \to 0 \qquad (13)$$

since equation (12) is linear (but with variable coefficients) subject to the boundary conditions (13), it is solved here numerically.

4.Numerical solution:

Now we begin with the development of the procedure for completing the numerical solution for ϕ(η). It is clear that $f''(0)$=-1.289747 in this problem by taking visco-elastic parameter k_1=0.2. Since the flow problem is uncoupled from the concentration problem, Changes in the values of Sc will not affect the fluid velocity. For this reason, both the function f and its derivatives are identical in the complete problem. In view of the above discussions, we have solved numerically, first the problem

{(5)-(6)} which provide $f''(0)$ and second, by with this result, we shall solve numerically Mass transfer problem. This procedure has already been applied to discuss some flow and heat transfer problem (Corttel.R[1993]). Equation (5) and (4) can easily be written as the first order system

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= u_4 \\ u_4' &= \frac{2k_1u_2u_4 - u_4 + u_2^2 - 2u_1u_3}{2k_1u_1} \\ u_5' &= u_6 \\ u_6' &= -\left(\frac{2}{3}\right)Scu_1u_6 + 2Scu_2u_5 \end{aligned} \tag{14}$$

Where the prime denotes differentiation with respect to η

u_1= f, u_5=ϕ and the value of $u_3(0) = f''(0)$ is given

Withal, in accordance with conditions (6) and (10) we obtain

$$u_1(0) = 0, u_2(0) = 1, u_3(0) = -1.289747, u_4(0) = \frac{1}{1-2k_1}, u_5(0) = 1 \tag{15}$$

$$u_2(\infty) = 0, u_5(\infty) = 0 \tag{16}$$

Using numerical methods of integration and disregarding temporarily the conditions (16), a family of solutions of {(14)-(15)} can be obtained for arbitrarily chosen values of u_6 (0). Tentatively we assume that a special values of $|\theta'(0)|$ yields a solution for which θ(η) and θ′ (η) vanishes at a certain η = η_∞ (condition (16)) and satisfies the additional condition

$$u_2(\eta_\infty) = 0, u_5(\eta_\infty) = 0 \tag{17}$$

We guess u_5 (0) and integrate equation (14) using condition (15) as an initial value problem by employing Runge-kutta algorithm for higher order initial value problems with the additional conditions (17).

5. Results and discussion:

Computation through employed numerical scheme has been carried out for various values of Schmidt number Sc for non-linear stretching surface. For illustration of the results numerical values are plotted in the form of graph. We notice from this graph that the effect of increasing the values of Sc leads to decrease the concentration profile ϕ(η) in the flow field. Physically, the increase of Sc means decrease of molecular diffusivity D, which results in decrease of concentration of boundary layer. Hence, the concentration of the species is higher for small values of Sc and lower for large values of Sc, i.e. species diffusion layer thickness is thinner for heavier particles (large values of Sc) than for lighter particles (smaller values of Sc). This result is in agreement with (Abel. M.S et al.[2002]). Computation has been carried out for wall concentration gradient $\phi_\eta(0)$. It displays the results as Schmidt number increases, concentration gradient $\phi_\eta(0)$ decreases.

6. Summary and Conclusions:

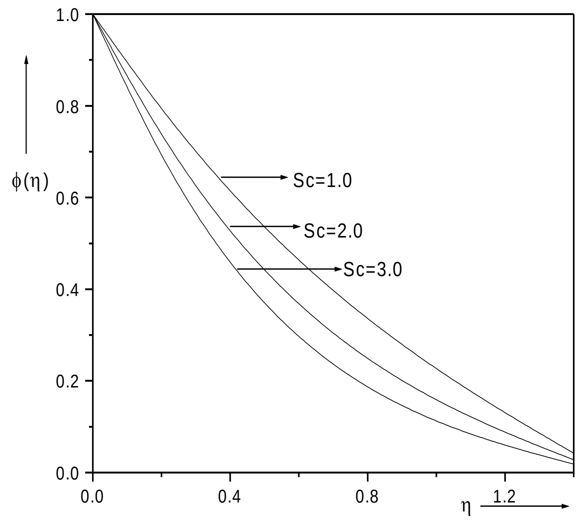
In this chapter we studied the influence of reaction rate parameter on the transfer of chemically reactive species in the laminar visco-elastic fluid flow for non-linearly stretching surface and the reactive species is emitted from this sheet and under goes an isothermal and homogeneous one-stage reaction as it diffuses in to the surrounding fluid. The mathematical problem has been solved numerically by shooting technique with fourth order Runge-Kutta integration scheme.

The important findings of our study are:

1) The thickness of the concentration boundary layer decreases with the increasing values of Schmidt number.
2) The thickness of the concentration boundary layer decreases with the increasing values of Schmidt number.
3) Increasing the values of Schmidt number, reaction rate parameter, wall concentration gradient $\phi_\eta(0)$ decreases.

Table: For different values Sc, Wall concentration gradient $\phi_\eta(0)$

Sc	$\phi_\eta(0)$
1.0	-0.838084
2.0	-1.053067
3.0	-1.213652



Variation of concentration profile $\phi(\eta)$ Vs η for different values of Scemidt number Sc,when k_1=0.2

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